# PROPERTIES OF EXPONENTS

***MULTIPLYING POWERS WITH LIKE BASES***

An **exponent** indicates how many times the base is a factor. In the expression $2^3$, the base is 2 and the exponent is 3. The exponent is indicating that the base 2 is a factor 3 times, that is $2 \cdot 2 \cdot 2$.

The expression $x^4 \cdot x^3$ can be expanded and simplified in the following way

$$x^4 \cdot x^3 = x \cdot x \cdot x \cdot x \;\cdot\; x \cdot x \cdot x = x^7$$

$x^4$ has four factors of $x$ and is being multiplied to $x^3$ which has three factors of $x$, so there is a total of seven factors of $x$.

***PRODUCT OF POWERS***

When multiplying two powers with the same base, add the exponents.

$$\boxed{\boldsymbol{x^m \cdot x^n = x^{m+n}}}$$

***Examples***: Simplify.

a) $x^{12} \cdot x^3$
$= x^{12+3}$
$= x^{15}$

b) $(2^{17} \cdot y^4)(2^{13} \cdot y^9)$
$= 2^{17+13} \cdot y^{4+9}$
$= 2^{30} \cdot y^{13}$

c) $(x+y)^7 (x+y)^2$
$= (x+y)^{7+2}$
$= (x+y)^9$

d) $b^{5/3} \cdot b^{1/3}$
$= b^{\left(\frac{5}{3}+\frac{1}{3}\right)}$
$= b^{6/3}$
$= b^2$

e) $(xy^6)(3x^2y^7)$
$= 3 \cdot x \cdot x^2 \cdot y^6 \cdot y^7$
$= 3 \cdot x^{1+2} \cdot y^{6+7}$
$= 3x^3y^{13}$

***EXERCISES:***

(1) $y^4y^6$

(2) $(2x^3y^4)\,(3x^5y^8)$

(3) $x^{3/5}\,x^{2/5}$

## ***DIVIDING POWERS WITH LIKE BASES***

The expression $\frac{x^5}{x^3}$ can be expanded and simplified in the following way:

$$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{\cancel{x \cdot x \cdot x} \cdot x \cdot x}{\cancel{x \cdot x \cdot x}} = \frac{x \cdot x}{1} = x^2$$

Note that the exponent of the result $x^2$ is the difference between the exponents of $x^5$ and $x^3$.

## ***QUOTIENT OF POWERS***

When dividing two powers with the same base, subtract the exponents.

$$\boxed{\frac{x^m}{x^n} = x^{m-n}}$$

***Examples***: Simplify.

a) $\frac{x^{12}}{x^3}$
$= x^{12-3}$
$= x^9$

b) $\frac{2^{17}y^9}{2^{13}y^4}$
$= 2^{17-13} \cdot y^{9-4}$
$= 2^4 y^5$
$= 16y^5$

c) $\frac{(x+y)^7}{(x+y)^2}$
$= (x+y)^{7-2}$
$= (x+y)^5$

d) $\frac{b^{4/3}}{b^{1/3}}$
$= b^{\left(\frac{4}{3}-\frac{1}{3}\right)}$
$= b^{3/3}$
$= b$

e) $\frac{3^2x^8y^{12}}{3x^2y^7}$
$= 3^{2-1} \cdot x^{8-2} \cdot y^{12-7}$
$= 3x^6y^5$

***EXERCISES:***

(4) $\frac{2^7x^{10}y^{15}}{2x^5y^7}$

(5) $\frac{t^3t^9}{t^4}$

(6) $\frac{(y-8)^9}{(y-8)^5}$

***RAISING A POWER TO A POWER***

The expression $(x^4)^3$ can be expanded and simplified in the following way:

$$(x^4)^3 = x^4 \cdot x^4 \cdot x^4 = x^{4+4+4} = x^{12}$$

Notice that the exponent of the result $(x^4)^3$ is the product of the powers 4 and 3.

***POWER OF A POWER***

When dividing two powers with the same base, subtract the exponents.

$$\boxed{(x^m)^n = x^{m \cdot n}}$$

***Example***: Simplify.

a) $(x^{12})^3$

$= x^{12 \cdot 3}$

$= x^{36}$

b) $(y^7)^2$

$= y^{7 \cdot 2}$

$= y^{14}$

c) $\left(b^{5/3}\right)^3$

$= b^{\left(\frac{5}{3} \cdot \frac{3}{1}\right)}$

$= b^{\left(\frac{5}{\cancel{3}} \cdot \frac{\cancel{3}}{1}\right)}$

$= b^5$

***EXERCISES:***

(7) $(x^3)^4$

(8) $(x^4)^6(x^2)^3$

(9) $\left(z^{1/3}\right)^{6/5}$

## ***RAISING A PRODUCT OR QUOTIENT TO A POWER***

The expression $(2x^4y)^3$ can be expanded and simplified the following way:

$$\begin{aligned}(2x^4y)^3 &= 2x^4y \cdot 2x^4y \cdot 2x^4y \\ &= 2 \cdot 2 \cdot 2 \cdot x^4 \cdot x^4 \cdot x^4 \cdot y \cdot y \cdot y \\ &= 2^{1+1+1} \cdot x^{4+4+4} \cdot y^{1+1+1} \\ &= 2^3 \cdot x^{12} \cdot y^3 \\ &= 8x^{12}y^3\end{aligned}$$

The factors of the product are $2, x^4$, and $y$. Notice that each factor was cubed, that is

$$(2x^4y)^3 = 2^3 \cdot (x^4)^3 \cdot y^3 = 8x^{12}y^3$$

### ***POWER OF A PRODUCT***

When dividing two powers with the same base, subtract the exponents.

$$\boxed{(xy)^n = x^n y^n}$$

The expression $\left(\frac{x^3}{y^2}\right)^3$ can be expanded and simplified the following way:

$$\begin{aligned}\left(\frac{x^3}{y^2}\right)^3 &= \frac{x^3}{y^2} \cdot \frac{x^3}{y^2} \cdot \frac{x^3}{y^2} \\ &= \frac{x^3 \cdot x^3 \cdot x^3}{y^2 \cdot y^2 \cdot y^2} \\ &= \frac{x^9}{y^6}\end{aligned}$$

Notice the numerator $x^3$ was raised to the third power, that is $(x^3)^3 = x^9$ and the denominator $y^2$ was also raised to the third power, $(y^2)^3 = y^6$.

### ***POWER OF A QUOTIENT***

When raising a quotient to a power, raise the numerator to the power and divide by the denominator to the power.

$$\boxed{\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}}$$

***Example***: Simplify.

a) $(xy)^3$

$= x^3 \cdot y^3$

$= x^3y^3$

b) $(2^8y^4)^6$

$= 2^{8\cdot6} \cdot y^{4\cdot6}$

$= 2^{54} \cdot y^{24}$

c) $\left(\frac{x}{y}\right)^4$

$= \frac{x^4}{y^4}$

d) $\left(\frac{2x}{y^4}\right)^3$

$= \frac{2^3\,x^3}{(y^4)^3}$

$= \frac{8x^3}{y^{12}}$

***EXERCISES:***

(10) $\left(\frac{c}{d^8}\right)^5$ (11) $\frac{\left(3x^4y\right)^3}{x^5}$ (12) $\frac{(6x)^5}{(6x)^3}$

***<u>EXPONENTS OF 0 AND 1</u>***

***<u>THE EXPONENT ONE</u>***

For any base $x$,

$$\boxed{\boldsymbol{x^1 = x}}$$

***<u>THE EXPONENT ZERO</u>***

A nonzero base raised to the 0 power is 1. For any nonzero base $x$,

$$\boxed{\boldsymbol{x^0 = 1}}$$

***Example***: Simplify.

a) $(x+2)^1$
$= x + 2$

b) $3^0$
$= 1$

c) $2(4x)^0$
$= 2 \cdot 1$
$= 2$

***EXERCISES:***

(13) $y^0$ (14) $(xy)^1(xy)^0$

***NEGATIVE EXPONENTS***

For any real number $x$ that is nonzero and any integer $n$,

$$\boldsymbol{x^{-n} = \frac{1}{x^n} \quad \text{and} \quad \frac{1}{x^{-n}} = x^n}$$

For any nonzero real numbers $x$ and $y$ and any integer $n$,

$$\boldsymbol{\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n}$$

where $x, y \neq 0$

***Example***: Simplify.

a) $x^{-3}$

$= \frac{1}{x^3}$

b) $2x^{-4}$

$= 2 \cdot x^{-4}$

$= 2 \cdot \frac{1}{x^4}$

$= \frac{2}{x^4}$

c) $\frac{3^{-2}}{x^4}$

$= \frac{1}{3^2 \cdot x^4}$

$= \frac{1}{9x^4}$

d) $\frac{1}{2^{-3}}$

$= 2^3$

$= 8$

e) $\frac{3x^{-3}}{x^{-2}}$

$= 3x^{[-3-(-2)]}$

$= 3x^{-1}$

$= \frac{3}{x}$

f) $\left(\frac{2x^2}{3y^{-3}}\right)^{-4}$

$= \left(\frac{3y^{-3}}{2x^2}\right)^4$

$= \frac{3^4 \cdot y^{-12}}{2^4 \cdot x^8}$

$= \frac{81}{16x^8y^{12}}$

***EXERCISES:***

(15) $x(y^3 \cdot y^{-3})$

(16) $\frac{5\,t^{-8}}{t^{-3}}$

(17) $(3x^3y)^{-2}$

(18) $\left(\frac{2x^2y^{-5}}{3x^0y^3}\right)^{-3}$

**Answers**

1.) $y^{10}$
2.) $6x^8y^{12}$
3.) $x$
4.) $2^6x^5y^8$
5.) $t^8$
6.) $(y-8)^4$
7.) $x^{12}$
8.) $x^{30}$
9.) $z^{2/5}$
10.) $\frac{c^5}{d^{40}}$
11.) $27x^7y^3$
12.) $36x^2$
13.) 1
14.) $xy$
15.) $x$
16.) $\frac{5}{t^5}$
17.) $\frac{1}{9x^6y^2}$
18.) $\frac{27y^{24}}{8x^6}$