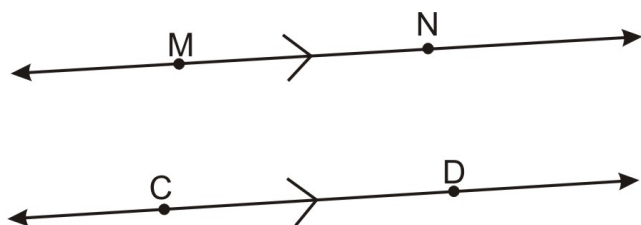


SUPPLEMENT 1B

1.3 Lines and Angles

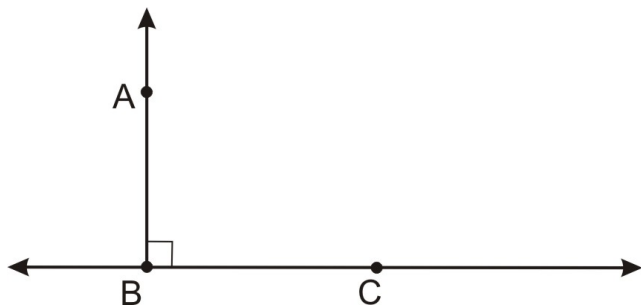
Parallel and Perpendicular Lines and Planes, and Skew Lines

Parallel lines are two or more lines that lie in the same **plane** and never intersect.



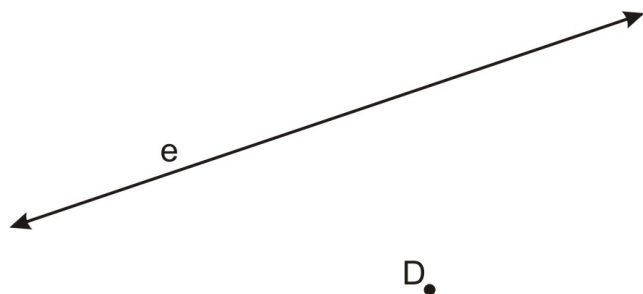
We use the symbol $\parallel$ for parallel, so to describe the figure above we would write $\overleftrightarrow{MN} \parallel \overleftrightarrow{CD}$. When we draw a pair of parallel lines, we use an arrow mark ($>$) to show that the lines are parallel. Just like with congruent segments, if there are two (or more) pairs of parallel lines, we use one arrow ($>$) for one pair and two (or more) arrows ($>>$) for the other pair.

Perpendicular lines intersect at a right angle. They form a 90° angle. This intersection is usually shown by a small square box in the 90° angle.

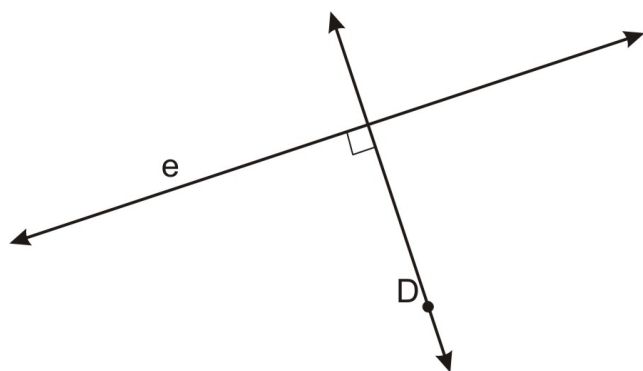


The symbol $\perp$ is used to show that two lines, segments, or rays are perpendicular. In the preceding picture, we could write $\overrightarrow{BA} \perp \overleftrightarrow{BC}$. (Note that $\overrightarrow{BA}$ is a ray while $\overleftrightarrow{BC}$ is a line.)

Note that although "parallel" and "perpendicular" are defined in terms of lines, the same definitions apply to rays and segments with the minor adjustment that two segments or rays are parallel (perpendicular) if the lines that contain the segments or rays are parallel (perpendicular).

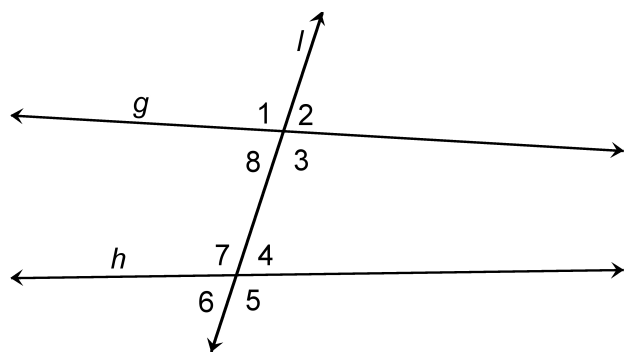


Remember that there can only be one line perpendicular to e that travels through point D . This line is drawn below.



Angles and Transversals

Many math problems involve the intersection of three or more lines. Examine the diagram below.



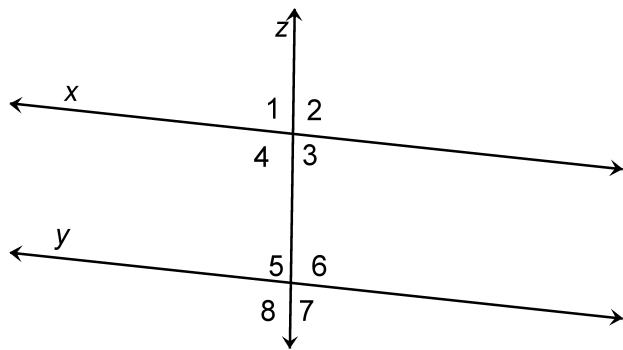
In the diagram, lines g and h are crossed by line l . We have quite a bit of vocabulary to describe this situation:

- Line l is called a **transversal** because it intersects two other lines (g and h). The intersection of line l with g and h forms eight angles as shown.

- The area between lines g and h is called the **interior** of the two lines. The area not between lines g and h is called the **exterior**.
- Angles $\angle 1$ and $\angle 2$ are called **adjacent angles** because they share a side and do not overlap. There are many pairs of adjacent angles in this diagram, including $\angle 2$ and $\angle 3$, $\angle 4$ and $\angle 7$, and $\angle 8$ and $\angle 1$.
- $\angle 1$ and $\angle 3$ are **vertical angles**. They are nonadjacent angles made by the intersection of two lines. Other pairs of vertical angles in this diagram are $\angle 2$ and $\angle 8$, $\angle 4$ and $\angle 6$, and $\angle 5$ and $\angle 7$.
- **Corresponding angles** are in the same position relative to both lines crossed by the transversal. $\angle 1$ is on the upper left corner of the intersection of lines g and l . $\angle 7$ is on the upper left corner of the intersection of lines h and l . So we say that $\angle 1$ and $\angle 7$ are corresponding angles.
- $\angle 3$ and $\angle 7$ are called **alternate interior angles**. They are in the interior region of the lines g and h and are on opposite sides of the transversal.
- Similarly, $\angle 2$ and $\angle 6$ are **alternate exterior angles** because they are on opposite sides of the transversal, and in the exterior of the region between g and h .
- Finally, $\angle 3$ and $\angle 4$ are **consecutive interior angles**. They are on the interior of the region between lines g and h and are next to each other. $\angle 8$ and $\angle 7$ are also consecutive interior angles.

Example 5

List all pairs of alternate angles in the diagram below.



There are two types of alternate angles—alternate interior angles and alternate exterior angles. As you need to list them both, begin with the alternate interior angles.

Alternate interior angles are on the interior region of the two lines crossed by the transversal, so that would include angles 3, 4, 5, and 6. Alternate angles are on opposite sides of the transversal, z . So, the two pairs of alternate interior angles are $\angle 3$ & $\angle 5$, and $\angle 4$ and $\angle 6$.

Alternate exterior angles are on the exterior region of the two lines crossed by the transversal, so that would include angles 1, 2, 8, and 7. Alternate angles are on opposite sides of the transversal, z . So, the two pairs of alternate exterior angles are $\angle 2$ & $\angle 8$, and $\angle 1$ and $\angle 7$.

Points to Consider

Parallel planes are two planes that do not intersect. Parallel lines must be in the same plane and they do not intersect. If more than two lines intersect at the same point and they are perpendicular, then they cannot be in same plane (e.g., the x -, y -, and z - axes are all perpendicular). However, if just two lines are perpendicular, then there is a plane that contains those two lines.

As you move on in your studies of parallel and perpendicular lines you will usually be working in one plane. This is often assumed in geometry problems. However, you must be careful about instances where you are working with multiple planes in space. Generally in three-dimensional space parallel and perpendicular lines are more challenging to work with.

Review Questions

Solve each problem.

1. Imagine a line going through each branch of the tree below (see the red lines in the image). What term best describes the two branches with lines in the tree pictured below?
2. How many lines can be drawn through point E that will be parallel to line m ?



3. Which of the following best describes skew lines?



Figure 3.4

- (a) They lie in the same plane but do not intersect.
 - (b) They intersect, but not at a right angle.
 - (c) They lie in different planes and never intersect.
 - (d) They intersect at a right angle.
4. Are the sides of the Transamerica Pyramid building in San Francisco parallel?
5. How many lines can be drawn through point M that will be perpendicular to line l ?

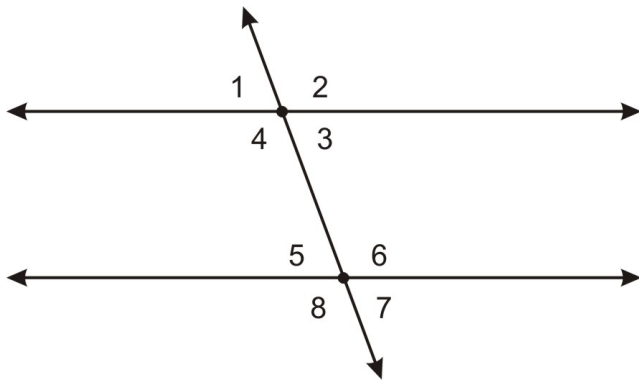


6. Which of the following best describes parallel lines?
- (a) They lie in the same plane but do not intersect.
 - (b) They intersect, but not at a right angle.
 - (c) They lie in different planes and never intersect.
 - (d) They intersect at a right angle.
7. Draw five parallel lines in the plane. How many regions is the plane divided into by these five lines?



Figure 3.5

8. If you draw n parallel lines in the plane, how many regions will the plane be divided into?



The diagram below shows two lines cut by a transversal. Use this diagram to answer questions 9 and 10.

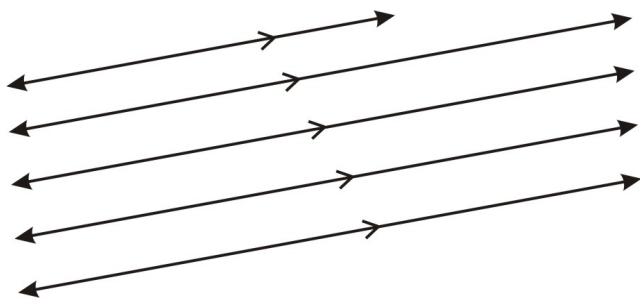
9. What term best describes the relationship between angles 1 and 5?
- (a) Consecutive interior
 - (b) Alternate exterior
 - (c) Alternate interior
 - (d) Corresponding

10. What term best describes angles 7 and 8?

- (a) Linear pair
- (b) Alternate exterior
- (c) Alternate interior
- (d) Corresponding

Review Answers

- 1. Skew
- 2. One
- 3. C
- 4. No
- 5. One
- 6. A
- 7. Five parallel lines divide the plane into six regions



- 8. n parallel lines divide the plane into $n + 1$ regions
- 9. D
- 10. A